Lecture 15: The Six-Factor Formula and Prompt Kinetics

CBE 30235: Introduction to Nuclear Engineering — D. T. Leighton

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Reading Assignment

Lamarsh & Baratta (4th Edition):

- **Chapter 7:** Time-Dependent Reactor Physics.
  - Sections 7.1 – 7.2: Reactor Kinetics and Delayed Neutrons.

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1 The "Bridge": From Buckling to the Six-Factor Formula

Over the last week, we analyzed reactors in two separate parts:

1. **The Physics (k_∞):** The Four-Factor Formula ($\epsilon p f \eta$) describes an infinite medium.
2. **The Geometry (Buckling):** The balance between Material Buckling (B_m^2) and Geometric Buckling (B_g^2) describes leakage.

In the CP-1 case study, we saw that Fermi needed to minimize leakage to reach criticality. We can formalize this by defining the **Effective Multiplication Factor**, k_{eff}.

1.1 The Six-Factor Formula

For a finite reactor, we multiply k_∞ by the probability that neutrons do *not* leak out. Since neutrons exist in two distinct energy groups (Fast and Thermal), we add two factors:

$$k_{eff} = k_\infty \cdot P_{FNL} \cdot P_{TNL} \tag{1}$$

Expanding k_∞:

$$k_{eff} = \epsilon p f \eta P_{FNL} P_{TNL} \tag{2}$$

1.2 Defining the Leakage Factors

Using the diffusion theory we derived:

1. Thermal Non-Leakage Probability (P_{TNL}): This is the probability a thermal neutron is absorbed in the core before diffusing out. From the one-group diffusion equation solution, this relates to the Geometric Buckling (B_g^2) and the Diffusion Length (L^2):

$$P_{TNL} = \frac{1}{1 + L^2 B_g^2} \tag{3}$$

Note: If the reactor is infinite, $B_g^2 \to 0$, so $P_{TNL} \to 1$.

2. Fast Non-Leakage Probability (P_{FNL}): Fast neutrons also migrate while slowing down. We define a new parameter, the **Fermi Age (τ_T)**. Despite the name "Age," it has units of area (cm^2), just like L^2. It represents (1/6) of the mean square distance a neutron travels from birth (fission) to thermalization.

$$P_{FNL} = e^{-B_g^2 \tau_T} \tag{4}$$

Approximation: For large reactors (small B_g^2), $e^{-x} \approx 1 - x$ or $1/(1 + x)$.

2 Introduction to Reactor Kinetics

So far, we have assumed $k_{eff} = 1$ exactly (Criticality). What happens if $k_{eff} \neq 1$? The neutron population $n(t)$ changes with time.

2.1 The Reactivity (ρ)

We define **Reactivity** as the fractional departure from criticality:

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{\Delta k}{k} \tag{5}$$

- $\rho = 0$: Critical ($k = 1$).
- $\rho > 0$: Super-critical.
- $\rho < 0$: Sub-critical.

3 Prompt Kinetics (The "Bomb" Approximation)

Let's assume for a moment that all neutrons are emitted instantly at the moment of fission. These are called **Prompt Neutrons**.

3.1 The Kinetic Equation

The rate of change of the neutron population $n(t)$ is:

$$\frac{dn}{dt} = \text{Production} - \text{Loss}$$

$$\text{Production} = k_{eff} \frac{n}{l}$$

$$\text{Loss} = \frac{n}{l}$$

Where l is the **Mean Prompt Neutron Generation Time** (lifetime of a prompt neutron from birth to death).

$$\frac{dn}{dt} = \frac{k_{eff} n}{l} - \frac{n}{l} = \frac{k_{eff} - 1}{l} n \tag{6}$$

3.2 The Solution

This is a simple first-order ODE.

$$n(t) = n_0 \exp\left(\frac{k_{eff} - 1}{l} t\right) \quad (7)$$

Or, identifying the **Reactor Period** $T = \frac{l}{k_{eff}-1}$:

$$n(t) = n_0 e^{t/T}$$

3.3 How Fast is it?

Let's plug in numbers for a thermal reactor.

- Mean Generation Time (l): $\approx 10^{-4}$ seconds (0.1 ms).
- Reactivity: Suppose we increase k by just 0.1% ($k = 1.001$).

$$T = \frac{10^{-4}}{0.001} = 0.1 \text{ seconds}$$

In just **1 second** ($t = 1$):

$$\frac{n(1)}{n_0} = e^{1/0.1} = e^{10} \approx 22,000$$

The power increases by a factor of 22,000 in one second.

In **10 seconds**, the factor is $e^{100} \approx 10^{43}$.

[Image of Exponential Growth Curve vs Linear]

Conclusion: If all neutrons were prompt, a nuclear reactor would be impossible to control mechanically. No human or machine could react fast enough to stop an excursion. It would effectively be a bomb. A bomb is prompt-supercritical without moderation and without delayed neutrons contributing. This is why prompt-supercritical systems behave like nuclear explosives.

4 The Saving Grace: Delayed Neutrons

Fortunately, not all neutrons appear instantly.

- $\approx$ 99.35% are **Prompt** (emitted within 10^{-14} s).
- $\approx$ 0.65% are **Delayed** (emitted seconds to minutes later).

This small fraction, denoted by β (beta), slows down the effective time constant of the reactor from milliseconds to seconds or minutes. (Note: this value for β is for U-235 thermal fission. For Pu-239 at high energies it is even smaller!)

4.1 The Precursors

Delayed neutrons come from the decay of fission products (called **Precursors**), like ^{87}Br.

$$^{87}\text{Br} \xrightarrow{T_{1/2}=55s} {}^{87}\text{Kr}^* \rightarrow {}^{86}\text{Kr} + n$$

The neutron isn't "born" until the Bromine decays. This introduces a "lag" in the system.

5 Summary

1. k_{eff} combines material properties (k_∞) and geometry (P_{NL}).
2. If $k_{eff} > 1$, power rises exponentially.
3. Based on prompt neutrons alone ($l \approx 10^{-4}$ s), the time scale is too fast to control.
4. **Next Lecture:** We will derive the "Inhour Equation" to see exactly how delayed neutrons (β) make nuclear power possible.